Einstein's Spacetime and Kant's Argument to Absolute Space

アインシュタインの時空とカントの絶対空間論

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Abstract

This paper examines Kant's 1768 argument from incongruent counterparts against relationism and for absolute space. I argue that abstract formal geometry and its associated concept of geometrical possibility allow a reply to the argument that avoids both naïve relationism and absolutism about spacetime, while connecting informatively to the modern conception of spacetime propounded by Einstein and Minkowski.

Keywords

enantiomorphs, incongruent counterparts, handedness, spacetime, absolute space, absolutism, relationalism, Kant, Einstein

Minkowski (1908), building on Einstein's theory of special relativity, showed how theories of kinematics can be presented as doctrines of spacetime structure. Einstein (1920) subsequently reconceptualized gravity as inertia within spacetime geometry. According to Einstein, spacetime is 'a structural quality of the physical field' ([1920] 1961, p.155) with no existence of its own, the bare spacetime manifold arising out of the way that physical fields (the metric fields of general relativity and electromagnetics) knit together (Stachel [1980] 1989). On this 'conventionalist' view, all the more striking for its apparent contrast with Einstein's notorious scientific realism, space is not an entity or a cause of patterns in our measurement operations: space is not 'absolute'.[1]

In 1768, Kant offered an argument that space must be absolute.

1　My understanding of Minkowski and Einstein owes a large debt to Philip Catton's lectures on scientific method at the University of Canterbury. Any inaccuracies are, of course, solely my responsibility.

Kant's 1768 argument from incongruent counterparts

Kant argues that 'the complete principle of determining a physical form does not rest merely on the relation and the situation of the parts, with respect to each other, but also on its relation to general absolute space, as conceived by geometers' (1768: II 381). His argument is from enantiomorphs or 'incongruent counterparts', mirror images which cannot be brought into superposition through continuous rigid motions (translations and rotations rather than reflections) — for example the two right-angled triangles formed by bisecting an isosceles triangle, or the right and left hands.

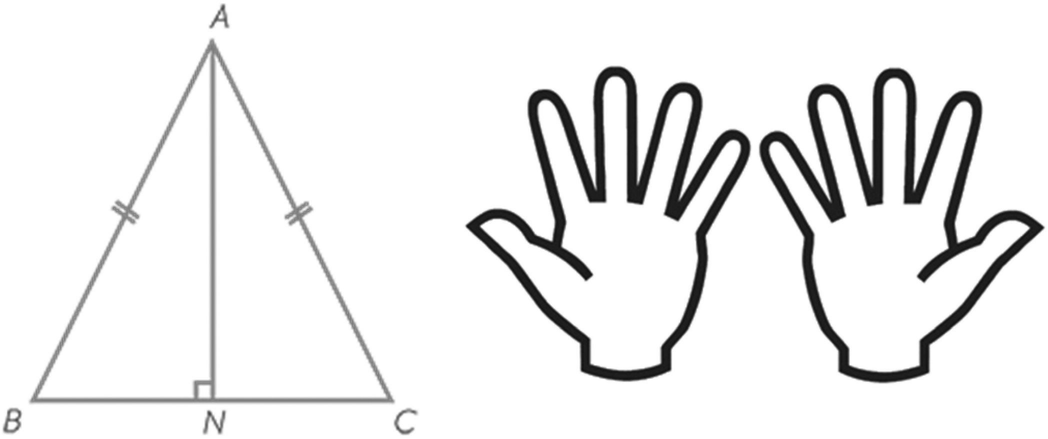

Figure: enantiomorphs (*the triangles ANB and ANC; left and right hands*)

Reconstructing Kant's argument, partly following van Cleve (1987: 34–5):

(1) Incongruent counterparts differ in some spatial respect (handedness) in virtue of differing interrelation of parts, or differing external relations to parts of matter (1768: II 383), or differing relations to absolute space.

(2) It cannot be in virtue of differing interrelation of parts, for 'a complete description of the one must apply, in all respects, to the other' (1768: II 381). For example, if ANB is a 'triangle with 3 and 4 cm sides meeting at 90 degrees' then so too must be ANC.

(3) It cannot be in virtue of differing external relations to parts of matter, since: (a) 'that the surface which includes the one could not possibly include the other' is an internal difference resting on some inner principle (1768: II 382) — presumably because a surface is just the limit of the extension of the object (1770: II 403n); (b) the difference would be present even if 'the first thing created were a human hand ... it must necessarily be either a right hand or a left hand. In order to produce the one a different action of the creative cause is necessary from that, by means of which its counterpart could be produced' (1768: II 382–3). Even if the only

object in the universe were a hand, it would be one of right or left handed.

(4) Incongruent counterparts differ in virtue of differing relations to absolute space.

(2) and (3) together constitute a rejection of relationalism. (1) states that absolutism is the only alternative, and (4) concludes that absolutism is correct. Absolute space, according to Kant, is some kind of *entity* whereby:

(a) the relations of matter are consequences of the determinations of space, rather than vice versa (1768: II 383). Geometrical properties of bodies are consequences of geometrical properties of the occupied space.

(b) 'absolute space has its own reality independently of the existence of all matter and ... is itself the ultimate foundation of the possibility of its composition' (1768: II 378). Space could be objectless.

(c) absolute space is not an 'object of external sensation'; 'we can only perceive through the relation to other bodies that which, in the form of a body, purely concerns its relation to pure space' (1768: II 383). Absolute space is not an object of experience, but we can experience differences in the geometrical properties of bodies which are consequences of differences in the occupied space.

So, according to Kant, we experience the differences between enantiomorphs, which are explained as being in virtue of their occupying different regions of space.

Kant's own later criticism of absolutism

In later works, Kant rejects absolutism, complaining that in conceiving of 'an absolute and boundless receptacle of possible things', it 'invents an infinite number of true relations without any entities related to one another' and thus 'pertains to the world of fable' (1770: II 403–4). In the *Critique*, he protests admitting 'two eternal and infinite self-subsistent non-entities (space and time) which are there (yet without there being anything real) only in order to contain in themselves all that is real' (A39/B 56). While it is unclear why he writes of 'nonentities' (perhaps he cannot imagine what kind of entity space would be), Kant is clearly concerned with the metaphysics, and perhaps epistemology, of absolute space.

In response to the incongruent counterparts argument, Kant gives a fourth alternative to the three options canvassed in (1): a *subjective* and *ideal* space which 'proceeds from the nature of the mind', making the geometrical axioms of space universally valid of experience (1770: II 403–4). This is developed in the first edition *Critique* (1781: A19–36), and by 1783 Kant was willing to redeploy the incongruent counterparts argument, this time concluding that space is not relationalist, but rather the 'mere form of our sensuous intuition' (1783: 285–6). However, the argument does not feature in the second edition *Critique*, where we find instead a reiteration of the argument that only ideal space allows synthetic *a priori* knowledge of geometry (1787: B40–2).

Thus, on Kant's later view, the difference between incongruent counterparts is made out not in terms of their differing relations with absolute space, but with cognizers (this theme was arguably prefigured in 1768 – see II 378, 380) – it cannot be discursively described, and 'can only be noticed by a certain act of pure intuition' (1770: II 403). This does not entail a rejection of (3): Kant may agree with (3a) that difference in handedness is an internal intuitive difference in the object of experience, and agree with (3b) that a lone hand must be determinate as to its handedness. The lone hand must of course be in relation to a point of view, a cognizer, for space is a feature of the world of appearances, which must be appearances *to someone*.

Modern geometry and handedness

Much remarked upon in commentary on Kant's argument is the development of geometries which are not necessarily three–dimensional, for if we allow n+1 dimensions, the n dimensional space will be non–orientable (Nerlich 1973: 343–5, Sklar 1974: 279, van Cleve 1987: 44–45). That is, two n–dimensional figures which are enantiomorphs when confined to n dimensions can be brought into superposition (that is, are homomorphs) if we allow continuous rigid motions through n+1 dimensions (in other words, a reflection in n dimensions can be achieved through a rotation in n+1 dimensions). For example, a right–angled triangle may have its handedness reversed by being moved around a Möbius strip (moved around a three–space–twisted two–space). However, the relevance of these developments must be carefully considered, for there is a difference between space and geometry. (Besides, we may be interested in local threespace not spacetime; and Kant's argument can presumably be rerun on the handedness of higher–dimensional objects.)

More relevantly, I believe, Kant's suggestion that the difference between right and left hands 'cannot be made intelligible by any concept' (1783: 286; see also 1770: II 403) have been shown to be false by subsequent advances in geometry. The determinant of the coefficient of non–enlarging linear transformations can be either +1 or –1, continuous rigid motions taking +1 and reflections taking –1. An object is 'handed' just if the class of possibilia which can be brought into congruence with it through continuous rigid motions are a proper subset of the class of possibilia which can be brought into congruence with it through non–enlarging linear transformations including reflections. Possibilia in the *proper subset* are of the same handedness as the object, and those only in the superset are of different handedness.

Which of these species is called 'right-handed' is a matter of convention — if the entire universe were to be transformed into its mirror-image, our use of 'left' and 'right' could continue subjectively unaltered; and if only I were left untransformed, the effect would be the same as only me being transformed (see for example Walker 1978: 50). In the terms introduced above, we may divide the objects which satisfy Kant's description from (2) into two species, H1 and H2. Each member of each species can be made congruous with any other member of its own species by transformations with determinant +1, but can be made congruous with a member of the other species only through transformations with determinant −1. A hand must be handed, but to ask which handedness it has is to ask after a convention: if the convention has not been fixed, the question may be answered with a coin-toss. Of course, once we have fixed the convention, our use must be consistent, and the question becomes substantive through its relation to other matters of fact. Then making out which hand it is requires information on which way the convention was decided — perhaps the demonstrative specification of an example of one species; or of the directional use of 'H1', followed by a description ('*this* direction is "H1", and a hand is "H1" just when, with the palm towards you and the fingers up for you, the thumb is on the H1 side'). This may be taken as some vindication of Kant's emphasis on a cogniser or a point of view — it is with essential reference to our practice, to how the convention was decided, that a hand's actual handedness is made out.

However, having made handedness intelligible geometrically, we may be able to reject the incongruent counterparts argument without following Kant in introducing a fourth alternative to the three options canvassed in (1). (2) seems unobjectionable, leaving (3). Against (3a), handedness is determined both by the object *and by global properties of the space* — dimensionality, orientability — which, relating to matter in general, does not seem to rest on an 'inner principle'. Further, handedness consists in relation to possibilia (transformations of the object in question), and thus seems to be a relational property rather than merely an 'inner principle'. But neither 'matter in general' nor possibilia are 'parts of matter', so (3) may stand while (3a) falls.

The relationship between matter in general, space, and these possibilia is exceedingly tight, for space is the geometry of matter in general, and the notion of 'possibility' in play here (even in (3a) itself) must relate to this geometry. Logic alone, being agnostic as regards dimensionality and metric etc., is too liberal — there is no *logical* contradiction in supposing any surface to possibly include the other. Physical possibility is too restrictive, for there may be non-geometrical bars to one surface including another — congruent objects at either end of an hourglass-shaped space would be incongruent if physical possibility were the relevant notion (see Sklar 1974: 282, Walker 1978: 52). The correct notion of possibility is, I suggest, (physical) *geometrical* possibility, a notion which is both conditioned by global patterns in the measurement of matter (by our empirical understanding of the geometry of spacetime) and less restrictive than the contingencies of physical possibility (congruent objects at either end of an hourglass–shaped space are by hypothesis geometrically congruent).

What of (3b), Kant's lone–hand thought experiment? A lone–hand must be handed, but to ask which of H1 and H2 it has is to ask after a convention － so if the hand is truly lone (we are not comparing it to another 'universe', or to a prior hand, as an observer with settled conventions), we may answer with a coin–toss. And we may agree with Kant that in 'order to produce the one a different action of the creative cause is necessary from that, by means of which its counterpart could be produced' － for an H2–handed hand is a (conceptually, geometrically) distinct object. So (3) may stand.

However, in both cases (3) stands only because it concerns 'relations to parts of matter' while handedness is in fact constituted by relations to possibilia: an object is 'handed' just if the class of possibilia which can be brought into congruence with it through transformations with determinant＋1 are a proper subset of the possibilia which can be brought into congruence with it through transformations with determinant＋1 or －1. So we need in fact to reject (1), as Kant did, but note 'relations to possibilia' (rather than 'relations to cognizers') as the further option.

What Kant's 1768 argument shows

Kant's 1768 argument 'from incongruent counterparts to absolute space' is therefore better construed as an argument 'from incongruent counterparts *against naive relationalism'* － (2) and (3) refute the view that space is no more than relations between matter, for we must consider at least also possibilia. Further, it militates against the view that all spatial concepts are *reducible* to relational concepts, for we need an abstract formal physical geometry to yield the possibilia in relation to which alone handedness can be explained.

Fortunately, this does not necessitate an acceptance of absolutism, for (as Kant came to see) the position involves metaphysical and epistemological problems (and has now been eschewed by most theorists working in the so–called foundations of physics).

Kant's later position － that incongruent counterparts *qua* appearances differ only in intuition, presumably underwritten by unknowable differences in the noumena (see Kant 1783: 286) － is an option if we are willing to accept transcendental idealism, but perhaps even then unnecessarily mystical, for we *can* give a conceptual characterization of the difference by making reference to possible objects, where 'possible' is made out in terms of (physical) geometrical possibility.

This connects to the modern conception of spacetime

This result, I believe, can usefully inform how to understand Einstein's position that spacetime is 'a structural quality of the physical field'.

Spacetime theory is that part of physical theory which is 'geometrical', where geometry concerns the constraints to which measurements will conform. That is, physicists discover elements reminiscent of geometry — dimensionality, continuity, and connectedness — in the formal apparatus used to describe the physical world.

Kant believed that further constraints — homogeneity, positive-definiteness — must also necessarily hold, and in this way underestimated the ways in which the geometrical parts of our physical theory are (like any other part of physical theory) subject to reconceptualization and review, though he was nonetheless arguably correct that there is something intuitive and inescapable for us perceptually or experientially in Euclidean geometry. In this way, we may simply accept Kant's complaint against relationalists that 'they throw down geometry from the summit of certitude and thrust it back into the rank of those sciences whose principles are empirical' (1770: II 404; also Critique A40/B56–7). For while geometrical proofs as formal exercises (or hypothetical physical geometries, which may approximate local spacetime) are non-empirical in that they draw their justification from the formal system itself, physical geometry, which concerns spacetime, is ultimately conditioned by physics *qua* empirical science, as shown by the discovery that spacetime is non-Euclidean.

However, this is not to expect a *reduction* of spatial geometry to relations between objects and an acceptance of naïve relationism, because we need the possibilia afforded by an abstract formal physical geometry — both of which outrun the actual relations between actual objects. We can agree with Kant, omitting only 'absolute', that 'the complete principle of determining a physical form does not rest merely on the relation and the situation of the parts, with respect to each other, but also on its relation to general ... space, as conceived by geometers' (1768: II 381). General space as conceived by geometers is a formal structure which outruns the relation and the situation of actual parts without being prior to those parts (without being absolute).

In terms familiar to physicists, spacetime being a structural quality of the physical field 'clearly does not imply that the content of spatial geometry somehow reduces to measurement operations. For Euclidean geometry systematises those measurements and exhibits them as aspects of a formal structure, something more abstract and more exact than the appearances could express by themselves' (DiSalle 1995: 323–324). In terms familiar to philosophers, spacetime is a formal structure rather than a causal one; is essentially epistemological (concerning the structure of our knowledge of what there is) rather than metaphysical or ontological.

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